Exact ray-tracing computation of narcissus equivalent temperature difference in scanning thermal imagers.

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ABSTRACT

The formula for evaluation of narcissus equivalent temperature difference as a function of the scan angle in thermal imaging systems is made more meaningful by grouping the parameters in two factors: one depending on wavelength and temperature and the other, a function of the scan angle, depending on the geometry of the instrument. Exact ray tracing equations are used to evaluate the ratio of radiant energy reaching the detector from warm and cold areas of the instrument.

1. INTRODUCTION

Discussion of the narcissus effect is often started by retelling the story of the eponymous unfortunate Greek mythological character. A detail neglected is that ever since to see one’s reflection in the water is considered as a presage of death. For the project engineer who discovers narcissus when the IR instrument is first tested at the optical bench it may not be that bad, after all the image displayed on the monitor screen is not his own, yet it may spell a great deal of trouble.

For these trivial reasons, and for serious reasons thoroughly discussed by other authors, evaluation of narcissus in IR scanners to a good approximation at the design stage is of utmost importance.

Numerical evaluation of narcissus, as all numerical analyses of physical problems, is based on approximations. To be able to interpret the results correctly it is necessary to understand the nature of the approximations. On this basis we discuss in some detail the derivation of the narcissus equivalent temperature difference equation.

A module for evaluation of narcissus along the lines described in this paper was incorporated in April 1992 into EIKONAL, the optical design software package developed by the authors for OCA Applied Optics.

2. NARCISSUS EFFECT IN THERMAL IMAGING SYSTEMS

Figure 2 shows a simplified thermal imaging system consisting of a telescope, a scanning mirror, an objective called imager and a row of detectors (normal to the plane of the figure) inside a cavity or dewar maintained at very low temperature. An aperture stop, called cold shield separates the cold cavity from the rest of the telescope. There are no optical elements between the cold shield and the
Every elementary area of the instrument inner walls, including the cold cavity, radiates energy in all directions. This energy propagates along rays. We assume that appropriate baffles and stops are designed into the instrument to prevent any of these rays to reach the detector elements directly.

Depending on the curvature and position of the lens surfaces in the optical system it is quite possible, however, that rays may reach the detector after reflection on one of them as shown in figures 3 and 4. Even though the amount of spurious energy reflected off a coated surface is very small the effect may be serious because the main signal received by the instrument comes from a low contrast object.

The detector array and associated electronic circuitry respond to the rate of change of energy, not to the extent of it, therefore we are concerned with the variation of the spurious signal that happens in the course of scanning.

In today's optical terminology the effect just described is called narcissus. The extreme difference of radiant energy reaching the detector from warm and cold instrument areas expressed in terms of temperature variation in the object space and called narcissus equivalent temperature difference (NETD) is a measure of this effect. Another useful measure is the narcissus equivalent temperature rate of change (NETRC), that is, the derivative of the former relative to scan angle. These two quantities are used to estimate to what extent narcissus affects the resolution of thermal images.
This discussion is limited to narcissus effect in a single detector element much in the same way that discussion of aberrations is restricted to point images. For finite detector arrays the results may be extended again in the same way as results of image analysis are extended from one point to a finite two-dimensional object by examining several critical points one at a time. In the case of images the combined effect is hardly ever integrated, but this possibility does exist in the case of a detector array.

It is assumed that both the scene and the instrument radiate as black bodies, it is valid therefore to use Planck’s radiation equation to express the dependence of irradiance on wavelength and absolute temperature. It is further assumed that the temperature is the same everywhere on the instrument walls except inside the cold cavity where the temperature is assumed to be zero degrees Kelvin. It is also assumed that the temperature difference in warmer and colder scene areas is small, but the mean scene temperature is not necessarily the same as the instrument temperature.

Absorption and Fresnel losses are neglected in the radiation reaching the detector from both the scene and the instrument warm and cold areas. Inter reflections and ghost images are also neglected. It is assumed that radiant energy propagates along rays and diffraction effects are neglected.

![Diagram](image)

**Figure 5. Elements involved in the derivation of NETD.**

### 4. DERIVATION OF NETD AND NETRC EQUATIONS

Figure 5 shows schematically the basic elements used to derive the NETD and NETRC equations. The optical system is represented by a single lens. A cold shield limits the radiation falling on the detector from any source. The object, or scene, is represented by a series of alternately warmer and colder than average vertical areas denoted by slightly different shades of gray suggesting small temperature differences. The cold cavity and warm instrument walls are represented by two large areas where the large difference in gray shading suggests large temperature differences.

#### 4.1. Basic radiometry equation and solid angle.

Radiant energy from the scene is refracted by lenses and focused on one of the elements of the detector array: an image is formed on the focal plane. Instrument walls energy may be received by the detector from many different areas and most likely so far out of focus that is unreal to speak even of an out-of-focus image. The radiometry problem of the narcissus spurious energy is more like the problem of a point on the floor receiving light from dark and bright areas on the ceiling through an aperture on a screen somewhere in between. Yet in either case (scene or instrument walls) the same radiometry equa-
equation applies: 
\[ H = \Omega N \]  
(1)

where \( H \) is irradiance, \( N \) is radiance and \( \Omega \) solid angle of the aperture subtended from the receiving area.

In the thermal imager let \( \Omega_A \) denote the solid angle of the cold shield aperture, \( A \) its area and \( R \) the distance to the detector, then
\[ \Omega_A = A / R^2 \]  
(2)

Since we assume that the scene and the instrument walls radiate as black bodies we can express radiance with Planck's equation:
\[ N_\lambda(T) = \frac{C_1}{\lambda^5} \left[ \exp \left( \frac{C_2}{\lambda T} \right) - 1 \right]^{-1} \]  
(3)

where \( \lambda \) is wavelength, \( T \) absolute temperature and \( C_1, C_2 \) are physical constants. In this discussion we need to use the partial derivative of Planck's equation with respect to the temperature:
\[ \frac{\partial N_\lambda(T)}{\partial T} = \frac{C_1 C_2}{\lambda^6 T^2} \exp \left( \frac{C_2}{\lambda T} \right) \left[ \exp \left( \frac{C_2}{\lambda T} \right) - 1 \right]^{-2} \]  
(4)

4.2 Scene radiance and irradiance differences

At any given angle in the scan cycle, since an image of the scene is formed on the focal plane, the cold shield aperture is filled with radiation from either a very small warmer or colder scene area. The irradiance on the scene image will be
\[ H_{WA} = \Omega_A N_\lambda(T_{WA}) \]  
(5)

from warmer scene areas, and
\[ H_{CA} = \Omega_A N_\lambda(T_{CA}) \]  
(6)

from colder scene areas. Subscripts WA, CA denote scene warm area and cold area, respectively.

The irradiance difference between the images of warmer and colder areas is
\[ \Delta H_S = \Omega_A \left[ N_\lambda(T_{WA}) - N_\lambda(T_{CA}) \right] \]  
(7)

If we define mean scene temperature and temperature difference using
\[ T_S = (T_{WA} + T_{CA})/2 \quad \Delta T_S = (T_{WA} - T_{CA}) \]  
(8)

and assume that temperature differences in the scene are not large we can expand equation (7) in a power series and neglect the terms of order higher than the first to obtain an expression for scene image irradiance difference in terms of scene temperature difference
\[ \Delta H_S \approx \Omega_A \frac{\partial N_\lambda(T_{MS})}{\partial T} \Delta T_S \]  
(9)

4.3. Instrument walls radiance and irradiance differences, fractional solid angle. Contribution of all surfaces in the system.

Even though there is no image formation by radiation coming from the instrument warm walls and cold cavity equation (1) can be used for the irradiance at the focal plane as explained earlier. Besides, the right hand side of the equation must be multiplied by \( r \), the reflectance coefficient of the
surface where reflection takes place to account for reflection losses. Then

\[ H_{WW} = r \omega_{WW} N_A(T_{WW}) \]  

(10)

is the focal plane irradiance produced by warm wall radiation and

\[ H_{CC} = r \omega_{CC} N_A(T_{CC}) \]  

(11)

is the focal plane irradiance produced by cold cavity radiation. The difference of equations (10) and (11) is

\[ \Delta H_W = r [\omega_{WW} N_A(T_{WW}) - \omega_{CC} N_A(T_{CC})] \]  

(12)

where

\[ \omega_{WW} = \frac{\omega_{WW}}{R^2} \quad \omega_{CC} = \frac{\omega_{CC}}{R^2} \]  

(13)

are solid angles subtended by the warm, and cold areas \( \omega_{WW}, \omega_{CC} \) respectively in the cold shield aperture at any given angle in the scan. We define a fractional warm solid angle as the ratio between the warm area solid angle and the cold shield area solid angle

\[ \sigma = \frac{\omega_{WW}}{\Omega_A} = 1 - \frac{\omega_{CC}}{\Omega_A} \]  

(14)

hence

\[ \omega_{WW} = \Omega_A \sigma(\psi) \]  

(15)

We have assumed now that the cold cavity temperature \( T_{CC} = 0 \), therefore

\[ \Delta H_W \approx r \omega_{WW} N_A(T_{WW}) \]  

(16)

We introduce here \( \sigma \) and make explicit the fact that this fractional solid angle, as well as the wall induced irradiance difference are functions of the scan angle by appending the notation "(\( \psi \))".

\[ \Delta H_W(\psi) = \Omega_A N_A(T_{WW}) r \sigma(\psi) \]  

(17)

We add together the wall induced irradiance difference contributed by each of \( J \) surfaces of the instrument:

\[ \sum_{j=1}^{J} \Delta H_W(\psi) = \Omega_A N_A(T_{WW}) \sum_{j=1}^{J} r_j \sigma_j(\psi) \]  

(18)

4.4. Equivalence of induced scene and wall irradiance differences. Extension to a finite bandwidth.

We want to find out the magnitude of the parameters involved in this derivation when the spurious narcissus signal interferes with the scene signal to a critical point, that is, when the narcissus signal and the scene signal are equal:

\[ \Delta H_S = \sum_{j=1}^{J} \Delta H_W(\psi) \]  

(19)

Replace equations (9) and (18) on each side of the equation above

\[ \Omega_A \frac{\partial N_A(T_S)}{\partial T} \Delta T_S = \Omega_A N_A(T_{WW}) \sum_{j=1}^{J} r_j \sigma_j(\psi) \]  

(20)
at a point on the wall of the cold cavity, and (2) if the ray never intersected the cold shield within the aperture it must have ended at a point on a warm wall, there is no other way.

The point where the scout ray ended is, then, a point on either a warm or a cold wall that radiates energy that travels an identical path as the scout ray but in opposite direction, is admitted by the cold shield aperture and reaches the detector. There is no need to verify this path.

A ray that does not end within the cold shield aperture does not end in an orderly manner. It may happen that: (1) the ray may exceed any clear aperture (including the cold shield, other aperture stops and field stops), (2) the ray angle of incidence on a surface may exceed the critical value (total internal reflection), (3) the ray may miss intersecting a lens surface. For these reasons it is better to keep track on the rays that end at the cold shield aperture.

Let $M$ be the total count of scout rays originated at the detector, equal to the number of elements into which the cold shield aperture is divided, and let $m$ the count of rays reflected at a selected lens surface and falling within the cold shield aperture. Then the number of rays falling outside will be $M - m$ and the fraction of solid angle $\sigma$ conveying warm energy from the wall to the detector is given by the ratio

$$\sigma = \frac{M - m}{M}$$

(25)

6. THE NARCISSUS ANALYSIS MODULE IN EIKONAL

The complete thermal imaging system to be analyzed (including the scanning mirror) is setup in the standard way, that is, front end pointing in the direction from where scene radiation is coming. It is important to check, before starting, that all clear apertures are properly defined, especially the cold shield aperture.

The user provides the following data: (1) List of selected surfaces where wall energy is partially reflected and reflectance coefficient for each surface, (2) Spectral band (short wavelength cut-off and long wavelength cut-off), (3) Scene average temperature (°K), (4) Instrument temperature (°K), (5) Cold shield aperture pattern (number of rows and columns), (6) Position (surface number) of scanning mirror in system, (7) Scan range (starting and ending angles in degrees) and (8) Number of samples in the scan range.

When the computations are completed the program returns on either hard copy or monitor screen results as in the example below.

Surface-by-surface narcissus contribution for each surface

<table>
<thead>
<tr>
<th>Surface No.2</th>
<th>reflectance 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANGLE</td>
<td>WARM ENERGY</td>
</tr>
<tr>
<td>deg.</td>
<td>%</td>
</tr>
<tr>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>1.000</td>
<td>16.67</td>
</tr>
<tr>
<td>2.000</td>
<td>33.33</td>
</tr>
</tbody>
</table>
3.000 50.63 0.260 0.086
4.000 66.04 0.339 0.079
5.000 81.76 0.420 0.081
6.000 94.97 0.488 0.068
7.000 100.00 0.513 0.026
8.000 100.00 0.513 0.000

Overall narcissus effect

<table>
<thead>
<tr>
<th>ANGLE deg.</th>
<th>NETD °K</th>
<th>NETRC °K/deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2.000</td>
<td>0.326</td>
<td>0.559</td>
</tr>
<tr>
<td>3.000</td>
<td>1.494</td>
<td>0.607</td>
</tr>
<tr>
<td>4.000</td>
<td>1.800</td>
<td>0.308</td>
</tr>
<tr>
<td>5.000</td>
<td>2.047</td>
<td>0.247</td>
</tr>
<tr>
<td>6.000</td>
<td>2.230</td>
<td>0.182</td>
</tr>
<tr>
<td>7.000</td>
<td>2.283</td>
<td>0.053</td>
</tr>
<tr>
<td>8.000</td>
<td>2.286</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Elsewhere in *EIKONAL* the product of the refractive index, height and angle of incidence of a paraxial marginal ray, \( \eta_i \), and ratio of angles of incidence of paraxial marginal and principal rays, \( \eta_{ibar} \), can be used as constraints to indirectly control narcissus at each of the system surfaces.

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8. REFERENCES